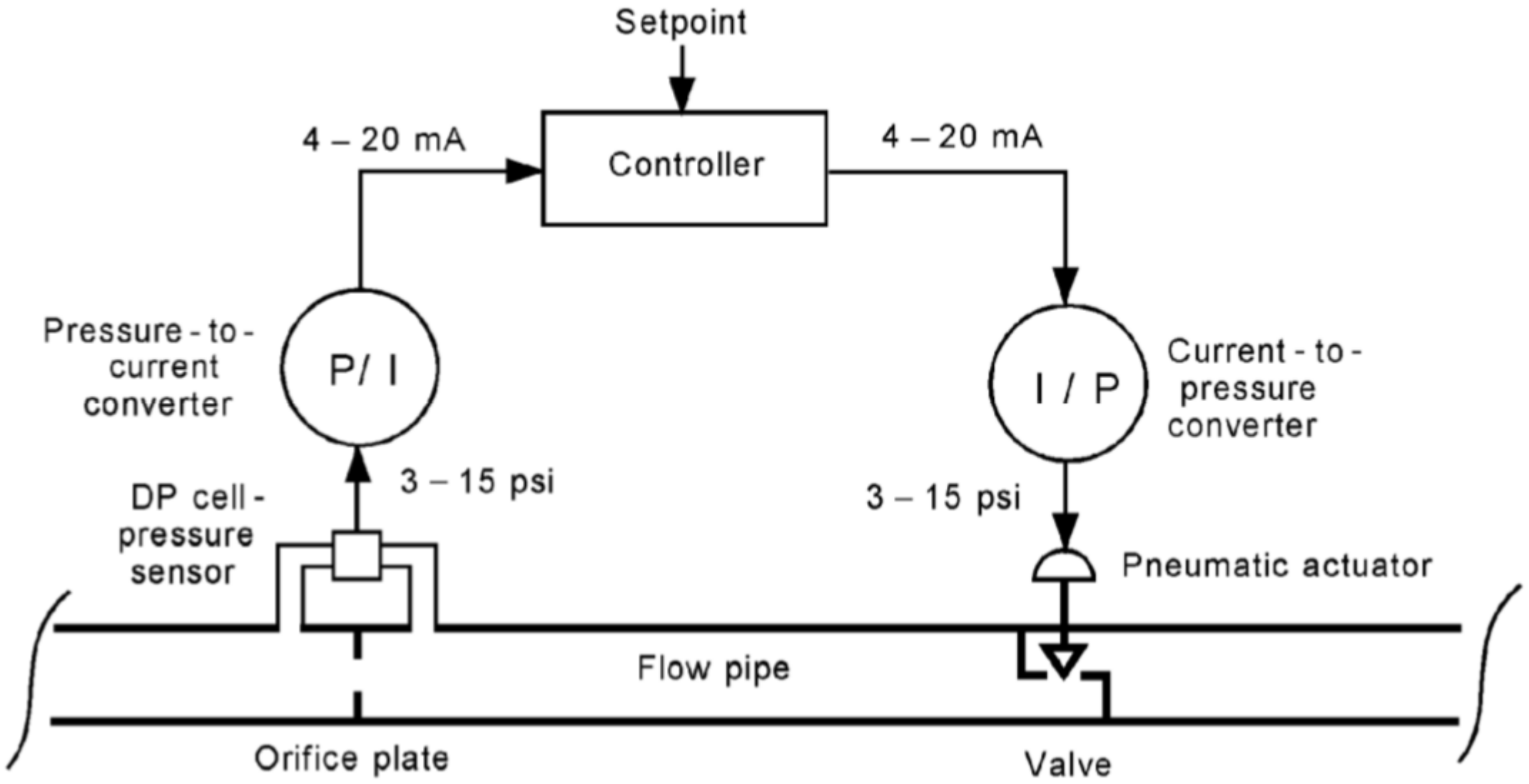
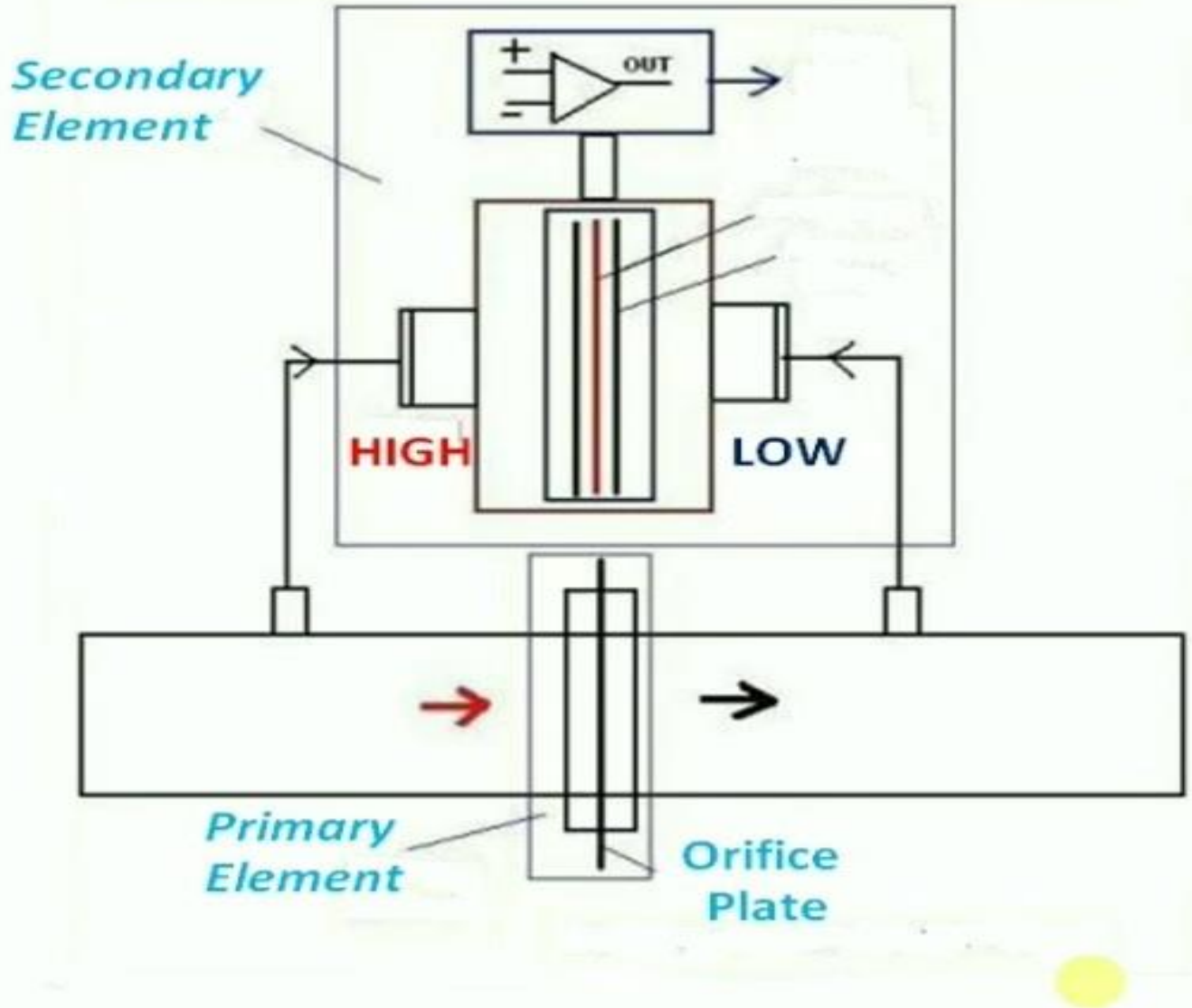
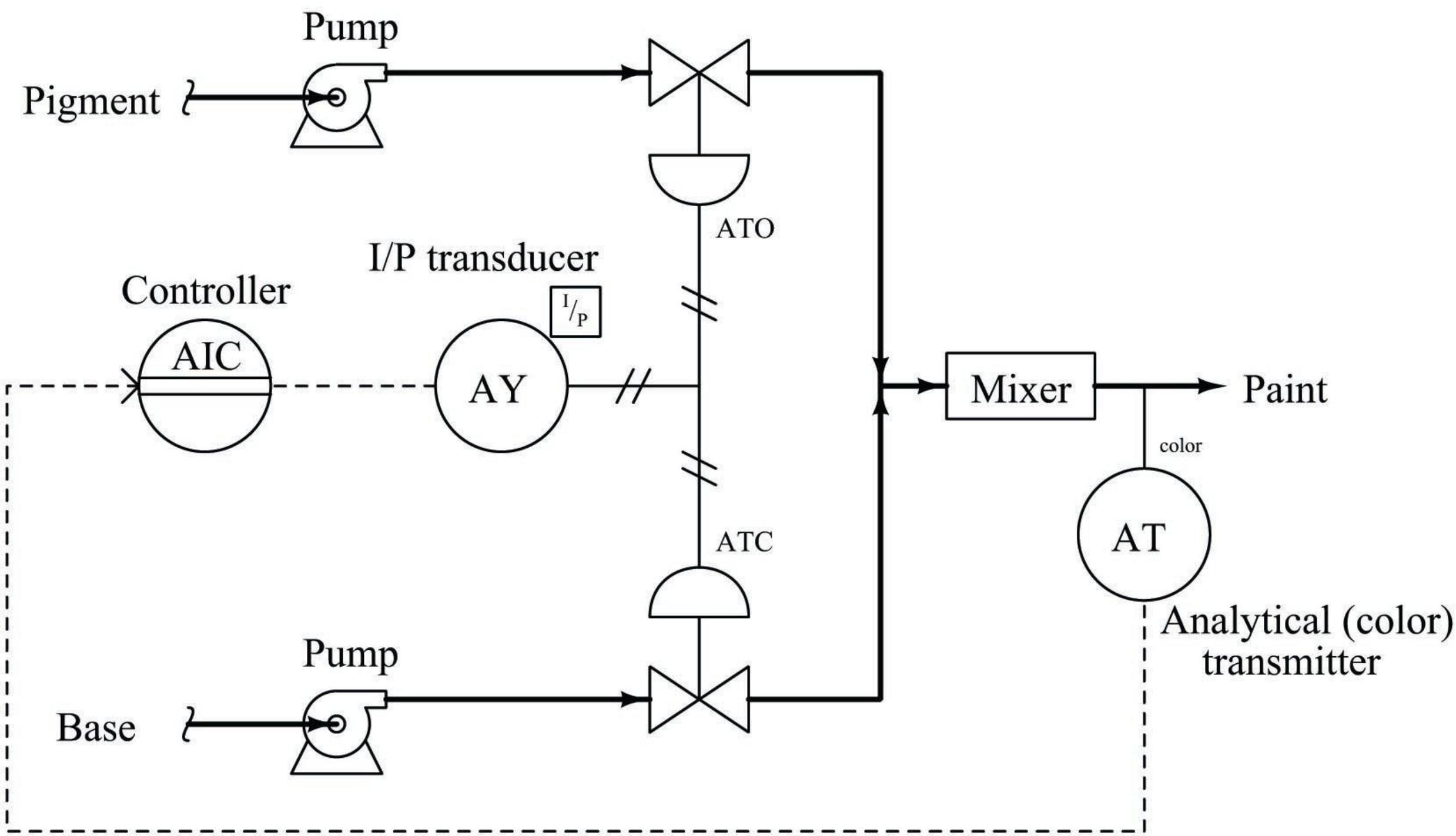
SENSORS & ACTUATORS (SESSION-4)

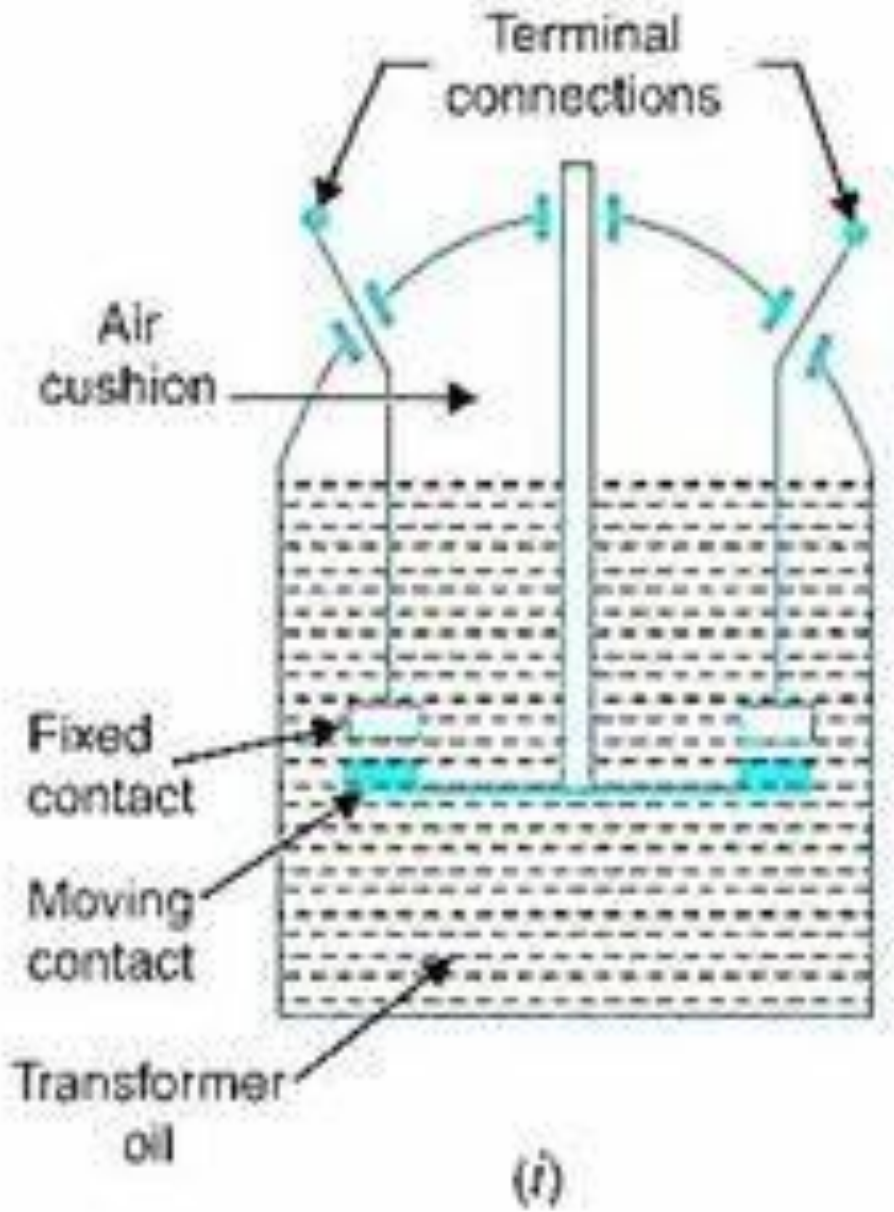
DR. ABDUL REHMAN ABBASI

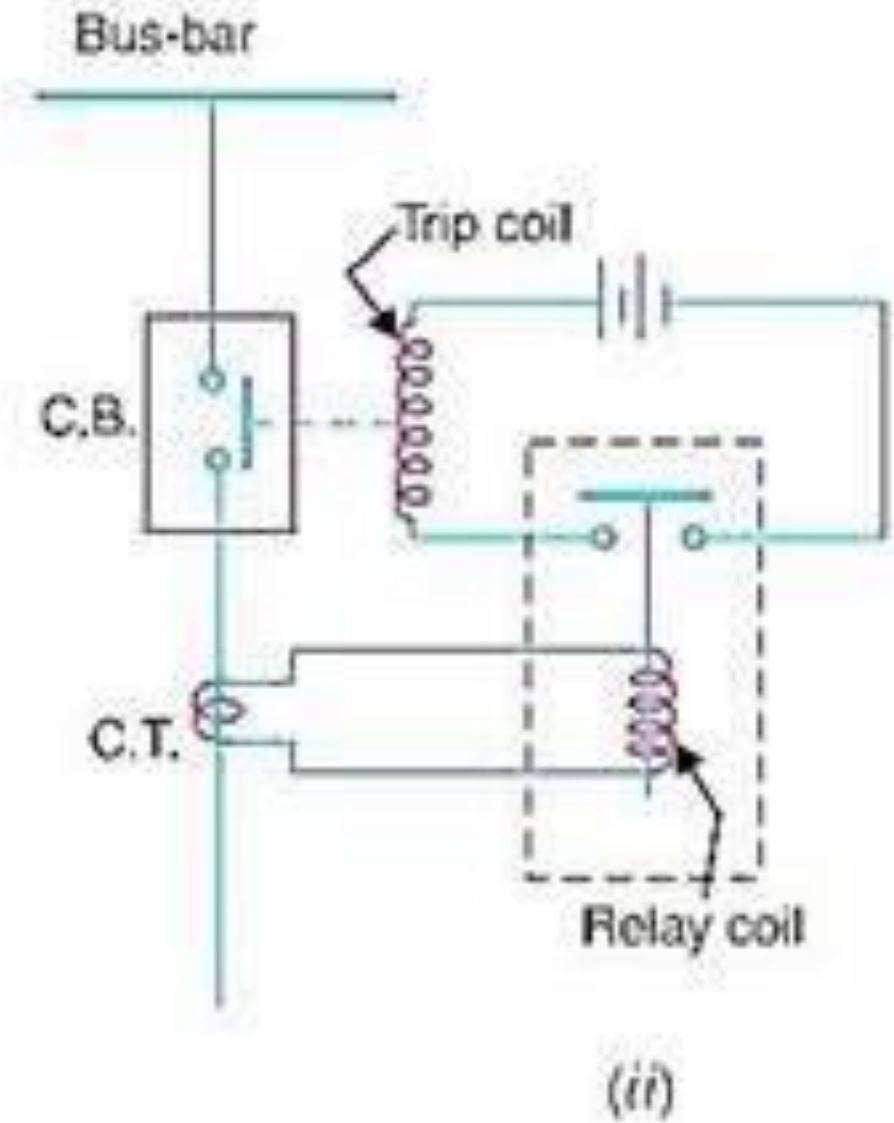

Setpoint
4 – 20 mA
Controller
4 – 20 mA
Pressure - to - current converter
P/ I
I / P
Current - to - pressure converter
3 – 15 psi
3 – 15 psi
DP cell - pressure sensor
Pneumatic actuator
Flow pipe
Orifice plate
Valve

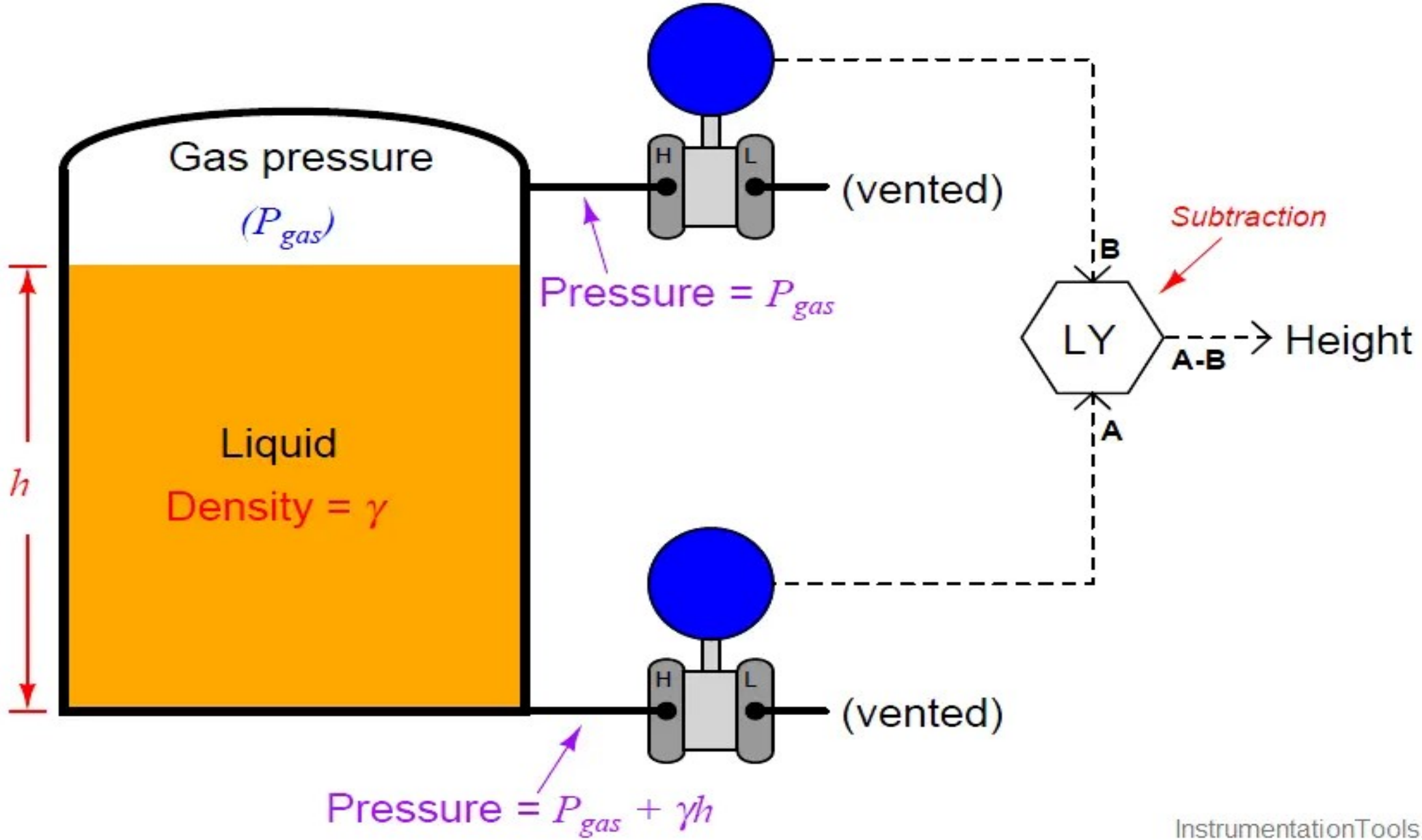
DIFFERNTIAL PRESSURE TRANSMITTER




Pump
Pigment
ATO
I/P transducer
Controller
I/P
AIC
AY
Mixer
Paint
color
AT
ATC
Pump
Base
Analytical (color)
transmitter


Terminal connections
Air cushion
Fixed contact
Moving contact
Transformer oil
(i)


Bus-bar
Trip coil
C.B.
C.T.
Relay coil
(ii)



InstrumentationTools.com

WIKA
14.60
PSI a
T/°F = 62.60
MAX 14.65

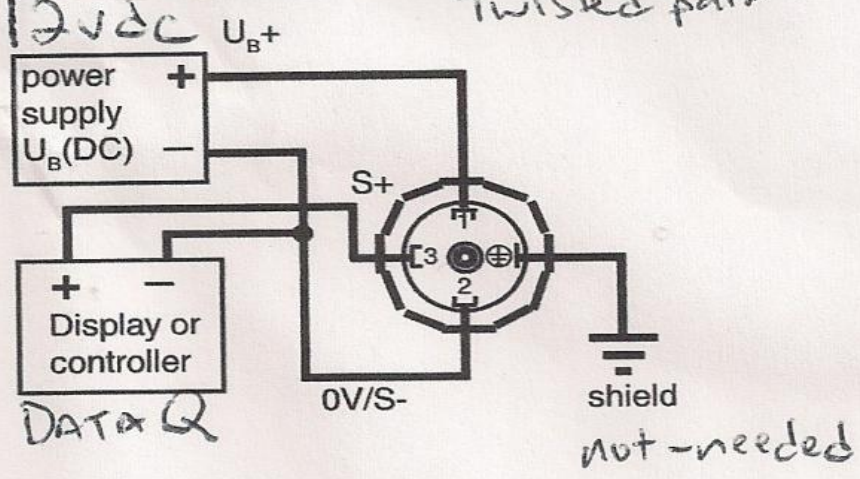
WIKA
Transmitter
0 ... 10000 psi
4 ... 20 mA
DC 10 ... 30 V
S # 4557881
P # 8357298

0-5V, 1-5V, 0.5-4.5V or 0-10V 3-wire system

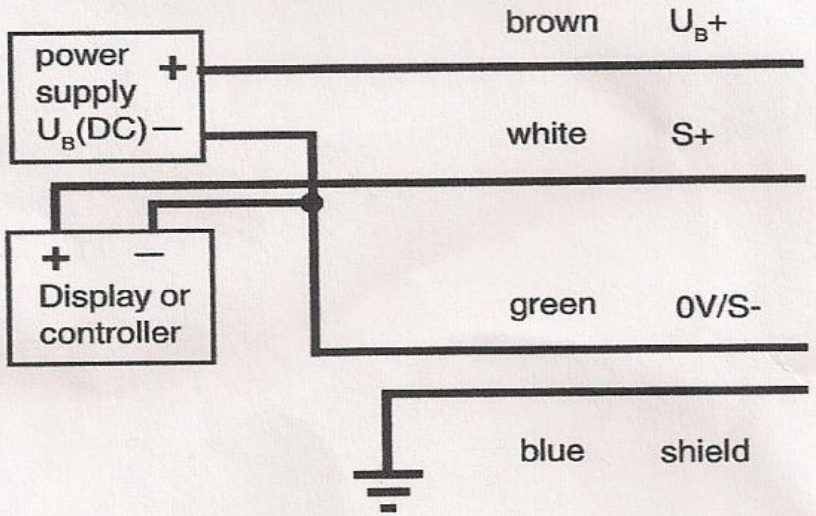
The 3-wire system features separate leads for the signal and power supply. The third lead is common minus for both devices. The signal source and indicating/recording instrument are connected in series, the power supply in parallel.

CAT 5 or 7
Twisted pair

DIN connector



Cable with free ends



Notes:

Terminal coding:

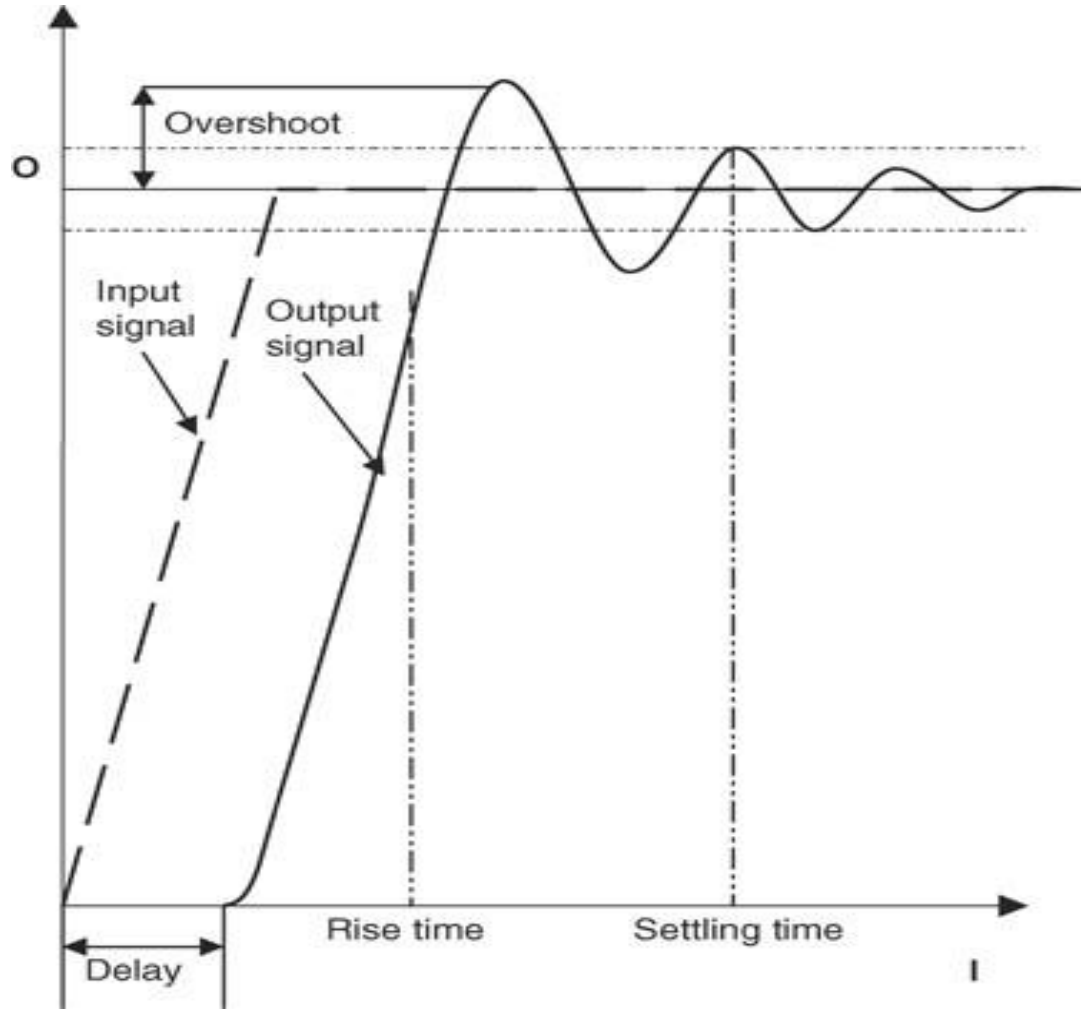
U_B+	Plus power supply
0V	Minus power supply (common, ground)
S +	Plus output signal
S -	Minus output signal (common, ground)
Shield	Cable shield / transmitter body

The supply voltage must be higher than the minimum required voltage as determined by the load equation for the specific transmitter. Refer to the specifications section of the data sheets for additional information.

SENSOR DYNAMIC CHARACTERISTICS

DYNAMIC CHARACTERISTICS OF SENSORS

- What really is a Dynamic Behavior of Sensors?
  - Behavior between the time a measured quantity changes value and the time when the instrument output attains a steady value in response.



DYNAMIC CHARACTERISTICS OF SENSORS

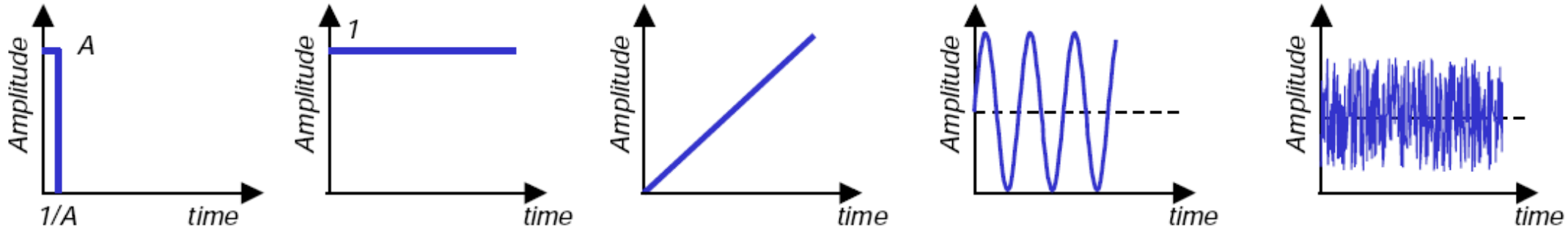
- Any idea HOW TO MODEL IT?

$$a_n \frac{\mathrm{d}^n q_0}{\mathrm{d}t^n} + a_{n-1} \frac{\mathrm{d}^{n-1} q_0}{\mathrm{d}t^{n-1}} + \cdots + a_1 \frac{\mathrm{d}q_0}{\mathrm{d}t} + a_0 q_0$$

$$= b_m \frac{\mathrm{d}^m q_i}{\mathrm{d}t^m} + b_{m-1} \frac{\mathrm{d}^{m-1} q_i}{\mathrm{d}t^{m-1}} + \cdots + b_1 \frac{\mathrm{d}q_i}{\mathrm{d}t} + b_0 q_i$$

DYNAMIC CHARACTERISTICS OF SENSORS

- Typical Inputs



- Reason for Dynamic Behavior?
  - Inertial: masses, inductances
  - Capacitances: electrical, thermal

DYNAMIC CHARACTERISTICS OF SENSORS

- For a step change (assuming a linear output)

$$a_n \frac{\mathrm{d}^n q_0}{\mathrm{d}t^n} + a_{n-1} \frac{\mathrm{d}^{n-1} q_0}{\mathrm{d}t^{n-1}} + \cdots + a_1 \frac{\mathrm{d}q_0}{\mathrm{d}t} + a_0 q_0$$
$$= b_m \frac{\mathrm{d}^m q_i}{\mathrm{d}t^m} + b_{m-1} \frac{\mathrm{d}^{m-1} q_i}{\mathrm{d}t^{m-1}} + \cdots + b_1 \frac{\mathrm{d}q_i}{\mathrm{d}t} + b_0 q_i$$

- Will reduce to

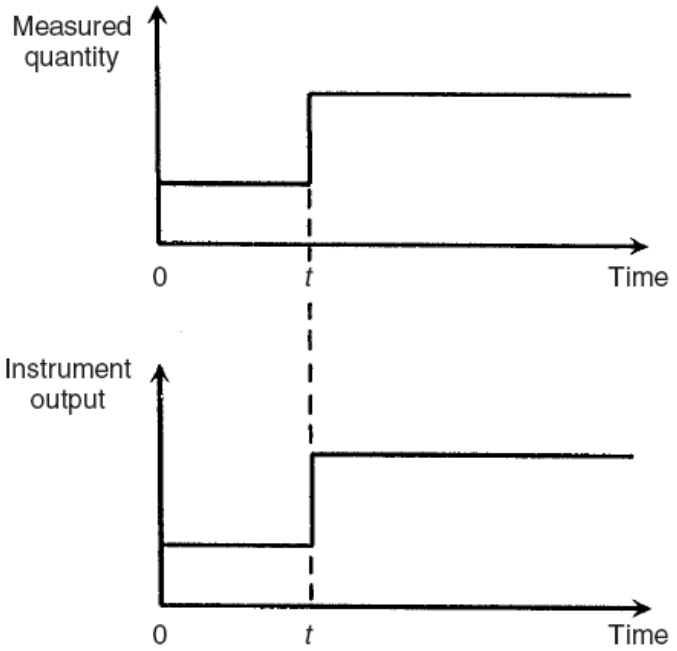
$$a_n \frac{\mathrm{d}^n q_0}{\mathrm{d}t^n} + a_{n-1} \frac{\mathrm{d}^{n-1} q_0}{\mathrm{d}t^{n-1}} + \cdots + a_1 \frac{\mathrm{d}q_0}{\mathrm{d}t} + a_0 q_0 = b_0 q_i$$

DYNAMIC CHARACTERISTICS OF SENSORS

$$a_n \frac{\mathrm{d}^n q_0}{\mathrm{d}t^n} + a_{n-1} \frac{\mathrm{d}^{n-1} q_0}{\mathrm{d}t^{n-1}} + \cdots + a_1 \frac{\mathrm{d}q_0}{\mathrm{d}t} + a_0 q_0 = b_0 q_i$$

- For Zero Order Instrument/Sensor:

$$a_0 q_0 = b_0 q_i \quad \text{or} \quad q_0 = b_0 q_i / a_0 = K q_i$$



Zero order instrument characteristic.

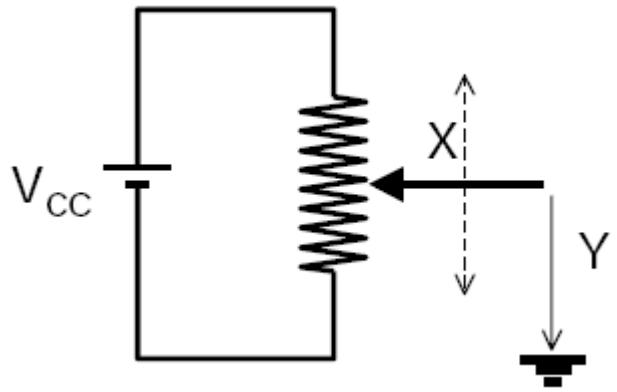
DYNAMIC CHARACTERISTICS OF SENSORS

$$a_n \frac{\mathrm{d}^n q_0}{\mathrm{d}t^n} + a_{n-1} \frac{\mathrm{d}^{n-1} q_0}{\mathrm{d}t^{n-1}} + \cdots + a_1 \frac{\mathrm{d}q_0}{\mathrm{d}t} + a_0 q_0 = b_0 q_i$$

- For Zero Order Instrument/Sensor:

$$a_0 q_0 = b_0 q_i \quad \text{or} \quad q_0 = b_0 q_i / a_0 = K q_i$$

- Example:



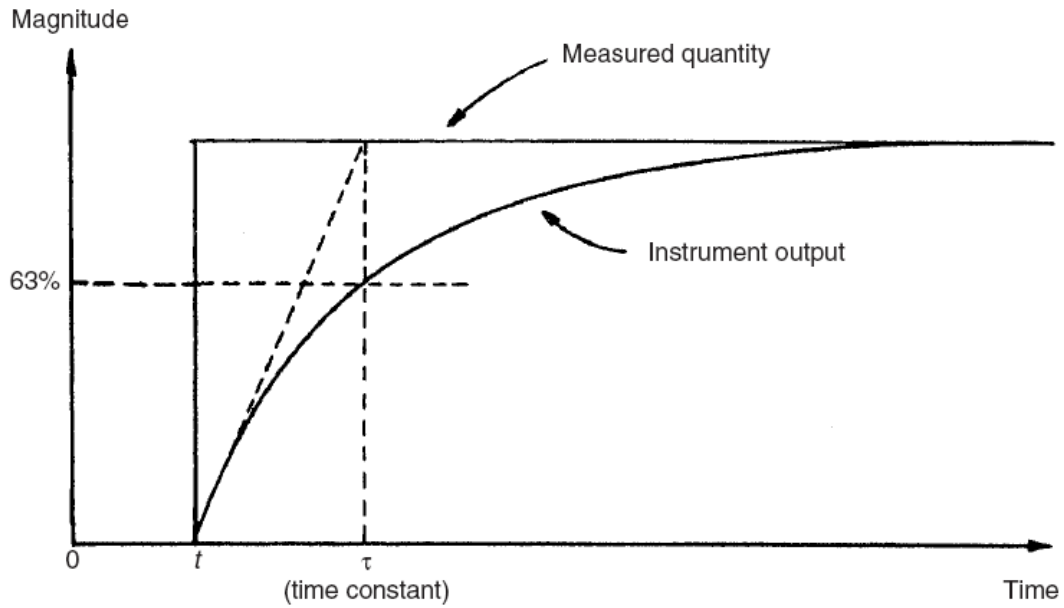
DYNAMIC CHARACTERISTICS OF SENSORS

$$a_1 \frac{dq_0}{dt} + a_0 q_0 = b_0 q_i$$

- First Order Sensors/ Instruments

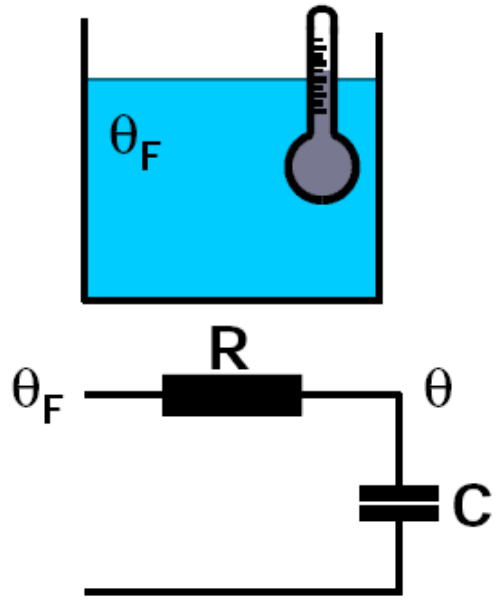
Defining $K = b_0/a_0$ as the static sensitivity and $\tau = a_1/a_0$ as the time constant of the system, equation (2.5) becomes:

$$q_0 = \frac{K q_i}{1 + \tau D} \qquad (2.6)$$



DYNAMIC CHARACTERISTICS OF SENSORS

- First Order Sensors/Instruments:
  - **A mercury thermometer immersed into a fluid**
    - What type of input was applied to the sensor?
    - Parameters
      - C: thermal capacitance of the mercury
      - R: thermal resistance of the glass to heat transfer
      - θ_F: temperature of the fluid
      - $\theta(t)$: temperature of the thermometer
    - The equivalent circuit is an RC network
  - **Derivation**
    - Heat flow through the glass $(\theta_F - \theta(t))/R$
    - Temperature of the thermometer rises as
    - Taking the Laplace transform



$$\frac{d\theta(t)}{dt} = \frac{\theta_F - \theta(t)}{RC}$$

FIRST ORDER DYNAMICS

A balloon is equipped with temperature and altitude measuring instruments and has radio equipment that can transmit the output readings of these instruments back to ground. The balloon is initially anchored to the ground with the instrument output readings in steady state. The altitude-measuring instrument is approximately zero order and the temperature transducer first order with a time constant of 15 seconds. The temperature on the ground, T_0, is 10°C and the temperature T_x at an altitude of x metres is given by the relation: $T_x = T_0 - 0.01x$

(a) If the balloon is released at time zero, and thereafter rises upwards at a velocity of 5 metres/second, draw a table showing the temperature and altitude measurements reported at intervals of 10 seconds over the first 50 seconds of travel. Show also in the table the error in each temperature reading.

(b) What temperature does the balloon report at an altitude of 5000 metres?

FIRST ORDER DYNAMICS

Let the temperature reported by the balloon at some general time t be T_r. Then T_x is related to T_r by the relation:

$$T_r = \frac{T_x}{1+\tau D} = \frac{T_0 - 0.01x}{1+\tau D} = \frac{10 - 0.01x}{1+15D}$$

It is given that $x = 5t$, thus: $T_r = \frac{10 - 0.05t}{1+15D}$

The transient or complementary function part of the solution ($T_x = 0$) is given by: $T_{r_{cf}} = Ce^{-t/15}$

The particular integral part of the solution is given by: $T_{r_{pi}} = 10 - 0.05(t - 15)$

Thus, the whole solution is given by: $T_r = T_{r_{cf}} + T_{r_{pi}} = Ce^{-t/15} + 10 - 0.05(t - 15)$

Applying initial conditions: At $t = 0$, $T_r = 10$, i.e. $10 = Ce^{-0} + 10 - 0.05(-15)$

Thus $C = -0.75$ and therefore: $T_r = 10 - 0.75e^{-t/15} - 0.05(t - 15)$

Using the above expression to calculate T_r for various values of t, the following table can be constructed:

Time	*Altitude*	*Temperature reading*	*Temperature error*
0	0	10	0
10	50	9.86	0.36
20	100	9.55	0.55
30	150	9.15	0.65
40	200	8.70	0.70
50	250	8.22	0.72

FIRST ORDER DYNAMICS

A balloon is equipped with temperature and altitude measuring instruments and has radio equipment that can transmit the output readings of these instruments back to ground. The balloon is initially anchored to the ground with the instrument output readings in steady state. The altitude-measuring instrument is approximately zero order and the temperature transducer first order with a time constant of 15 seconds. The temperature on the ground, T_0, is 10°C and the temperature T_x at an altitude of x metres is given by the relation: $T_x = T_0 - 0.01x$

(a) If the balloon is released at time zero, and thereafter rises upwards at a velocity of 5 metres/second, draw a table showing the temperature and altitude measurements reported at intervals of 10 seconds over the first 50 seconds of travel. Show also in the table the error in each temperature reading.
(b) What temperature does the balloon report at an altitude of 5000 metres?

(b) At 5000 m, $t = 1000$ seconds. Calculating T_r from the above expression:

$$T_r = 10 - 0.75e^{-1000/15} - 0.05(1000 - 15)$$

The exponential term approximates to zero and so T_r can be written as:

$$T_r \approx 10 - 0.05(985) = -39.25^{\circ}\text{C}$$

2ND ORDER INSTRUMENTS/SENSORS

$$a_n \frac{\mathrm{d}^n q_0}{\mathrm{d}t^n} + a_{n-1} \frac{\mathrm{d}^{n-1} q_0}{\mathrm{d}t^{n-1}} + \cdots + a_1 \frac{\mathrm{d}q_0}{\mathrm{d}t} + a_0 q_0$$

$$= b_m \frac{\mathrm{d}^m q_i}{\mathrm{d}t^m} + b_{m-1} \frac{\mathrm{d}^{m-1} q_i}{\mathrm{d}t^{m-1}} + \cdots + b_1 \frac{\mathrm{d}q_i}{\mathrm{d}t} + b_0 q_i$$

$$a_2 \frac{\mathrm{d}^2 q_0}{\mathrm{d}t^2} + a_1 \frac{\mathrm{d}q_0}{\mathrm{d}t} + a_0 q_0 = b_0 q_i$$

$$\frac{q_0}{q_i} = \frac{K}{D^2/\omega^2 + 2\xi D/\omega + 1}$$

$$K = b_0/a_0; \quad \omega = a_0/a_2; \quad \xi = a_1/2a_0a_2$$

2ND ORDER INSTRUMENTS/SENSORS

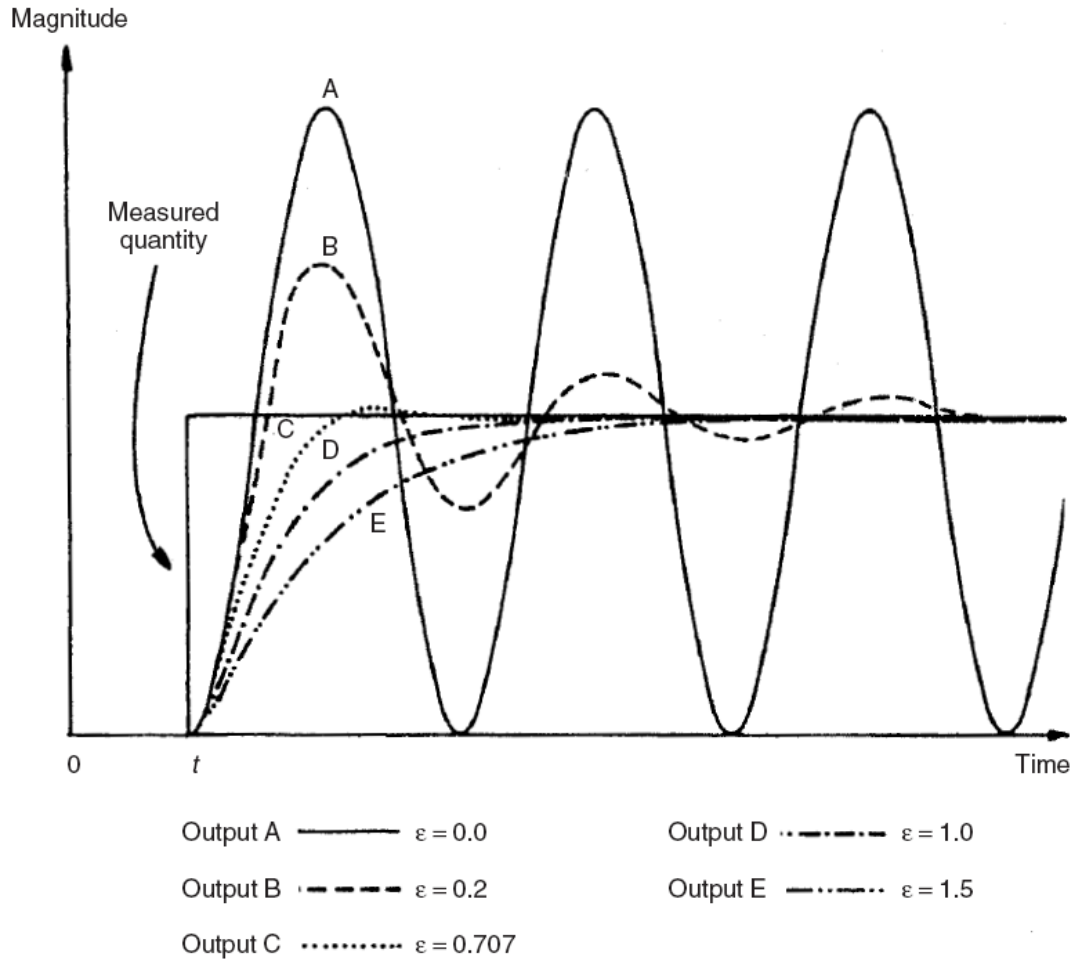
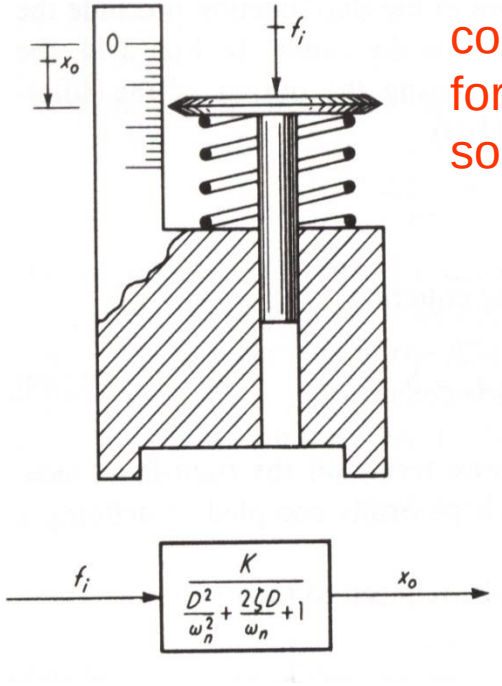


Fig. 2.12 Response characteristics of second order instruments.

2nd Order Sensor Dynamics: Example

Example: The force-measuring spring



consider a spring with spring constant K_s under applied force f_i and the total mass M. At start, the scale is adjusted so that $x_o = 0$ when $f_i = 0$;

$$\Sigma\text{forces} = (\text{mass})(\text{acceleration})$$

$$f_i - B\frac{dx_o}{dt} - K_s x_o = M\frac{d^2x_o}{dt^2}$$

$$(MD^2 + BD + K_s)x_o = f_i$$

the second-order model:

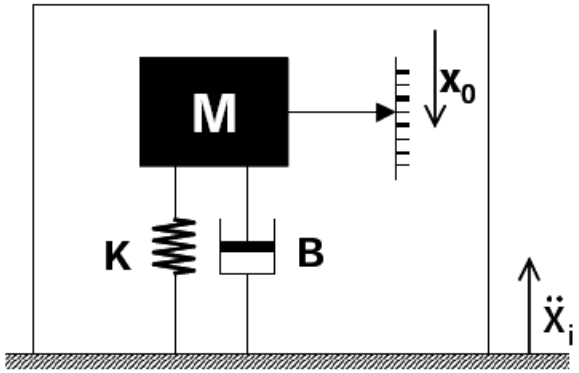
$$K = \frac{1}{K_s} \quad \text{m/N}$$

$$\omega_n = \sqrt{\frac{K_s}{M}} \quad \text{rad/s}$$

$$\zeta = \frac{B}{2\sqrt{K_s M}}$$

2ND ORDER SENSOR DYNAMICS: EXAMPLES

- **A thermometer covered for protection**
  - Adding the heat capacity and thermal resistance of the protection yields a second-order system with two real poles (overdamped)
- **Spring-mass-dampen accelerometer**
  - The armature suffers an acceleration
    - We will assume that this acceleration is orthogonal to the direction of gravity
  - x_0 is the displacement of the mass M with respect to the armature
  - The equilibrium equation is:



$$M(\ddot{x}_i - \ddot{x}_0) = Kx_0 + B\dot{x}_0$$

$$\Downarrow$$

$$Ms^2X_i(s) = X_0(s)\left[K + Bs + Ms^2\right]$$

$$\Downarrow$$

$$\frac{X_0(s)}{s^2X_i(s)} = \frac{M}{K}\frac{K/M}{s^2 + s(B/M) + K/M}$$

SELF-TEST PROBLEM

An unmanned submarine is equipped with temperature and depth measuring instruments and has radio equipment that can transmit the output readings of these instruments back to the surface. The submarine is initially floating on the surface of the sea with the instrument output readings in steady state. The depth-measuring instrument is approximately zero order and the temperature transducer first order with a time constant of 50 seconds. The water temperature on the sea surface, T_0, is 20°C and the temperature T_x at a depth of x metres is given by the relation:

$$T_x = T_0 - 0.01x$$

(a) If the submarine starts diving at time zero, and thereafter goes down at a velocity of 0.5 metres/second, draw a table showing the temperature and depth measurements reported at intervals of 100 seconds over the first 500 seconds of travel. Show also in the table the error in each temperature reading.
(b) What temperature does the submarine report at a depth of 1000 metres?